تفکیک کسر(عبارت های گویا)

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$\int \frac{f(x)}{g(x)}$ چندجمله‌ای از درجه n ، $g(x)$ " " از درجه m

$a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0 \qquad a_0, a_1, \dots, a_n \in \mathbb{R}$

$n<m$ $\qquad g(x)$ قابل تجزیه $\qquad \boxed{ax^2+bx+c \quad \Delta<0}$

$x-a \qquad \frac{A}{x-a}$

$(x-a)^k \rightsquigarrow$ ریشه مکرر (k مرتبه) $\quad \underbrace{\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \dots + \frac{A_k}{(x-a)^k}}_{k \text{ تا}}$

x^2+a^2

یا ($ax^2+bx+c \quad \Delta<0$) $\qquad \frac{Ax+B}{x^2+a^2}$

$$\int \frac{1}{x}dx = \ln|x| \qquad \int \frac{dx}{x\pm a} = \ln|x\pm a|+C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+C \qquad \int \frac{x}{x^2+a^2}dx = \frac{1}{2}\ln(x^2+a^2)+C$$

$$\int \frac{dx}{(\underbrace{x\pm a}_{u})^k} = \int (x\pm a)^{-k}dx = \frac{(x\pm a)^{-k+1}}{-k+1}+C$$

1) $\int \frac{2x-5}{x^2-9}dx \qquad \frac{2x-5}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3} = \frac{A(x+3)+B(x-3)}{(x-3)(x+3)}$

$(x-3)(x+3)$ $\qquad$ صورت کسر: $Ax+3A+Bx-3B = (A+B)x+(3A-3B) = 2x-5$

$3A+3B=6$

$$\begin{cases} A+B=2 \\ 3A-3B=-5 \end{cases} \rightarrow 6A=1 \rightarrow A=\frac{1}{6} \qquad A+B=2 \Rightarrow B=2-A=2-\frac{1}{6}=\frac{11}{6}$$

$$\Rightarrow \int \frac{2x-5}{x^2-9}dx = \int\left(\frac{1/6}{x-3} + \frac{11/6}{x+3}\right)dx = \frac{1}{6}\int\frac{1}{x-3}dx + \frac{11}{6}\int\frac{1}{x+3}dx$$

$$= \underbrace{\frac{1}{6}\ln|x-3| + \frac{11}{6}\ln|x+3|}_{\frac{1}{6}(\ln|x-3| + \ln|x+3|^{11})} + C = \frac{1}{6}\ln|x-3|.|x+3|^{11} + C$$

$$= \ln\sqrt[6]{|x-3|.|x+3|^{11}} + C$$

2) $\int \frac{dx}{x(x+2)^2}$

$$\frac{1}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{A(x+2)^2+Bx(x+2)+Cx}{x(x+2)^2}$$

صورت کسر : $A(x^2+4x+4) + Bx^2+2Bx+Cx = Ax^2+4Ax+4A+Bx^2+2Bx+Cx$

$=(A+B)x^2+(4A+2B+C)x+4A=1$

$$\begin{cases} A+B=0 \rightarrow B=-\tfrac{1}{4} \\ 4A+2B+C=0 \rightarrow 1-\cancel{\tfrac{1}{4}}\,^{\tfrac{1}{2}}+C=0 \Rightarrow C=-\tfrac{1}{2} \\ 4A=1 \rightarrow A=\tfrac{1}{4} \end{cases}$$

$$\Rightarrow \int \frac{dx}{x(x+2)^2} = \int\left(\frac{1/4}{x}-\frac{1/4}{x+2}-\frac{\frac{1}{2}}{(x+2)^2}\right)dx =$$

$$\int u^{-2}du = -u^{-1} = -\tfrac{1}{u} = -\tfrac{1}{x+2}$$

$$= \underbrace{\tfrac{1}{4}\ln|x| - \tfrac{1}{4}\ln|x+2|}\ - \tfrac{1}{2}\int \frac{\overset{du}{dx}}{(\underset{u}{x+2})^2}$$

$$= \tfrac{1}{4}\ln\left|\frac{x}{x+2}\right| + \frac{1}{2(x+2)} + C$$

3) $\int \dfrac{dx}{x^3+4x}$ $\qquad x^3+4x \rightarrow x(x^2+4)$

$$\frac{1}{x(x^2+4)} = \frac{A}{x}+\frac{Bx+C}{x^2+4} = \frac{A(x^2+4)+(Bx+C)x}{x(x^2+4)}$$

صورت کسر: $Ax^2+4A+Bx^2+Cx=(A+B)x^2+Cx+4A=1$

$A+B=0 \rightarrow B=-\tfrac{1}{4}$ $\qquad C=0$ $\qquad 4A=1 \rightarrow A=\tfrac{1}{4}$

$$\Rightarrow \int\frac{dx}{x^3+4x} = \int\left(\frac{1/4}{x}-\frac{\frac{1}{4}x}{x^2+4}\right)dx = \tfrac{1}{4}\left\{\int\left(\tfrac{1}{x}-\tfrac{1}{2}\cdot\frac{2x}{x^2+4}\right)dx\right\}$$

$$= \tfrac{1}{4}\ln|x| - \tfrac{1}{8}\ln(x^2+4) + C$$

4) $\int \dfrac{dx}{(x^2+1)(x^2+2)}$ $\qquad \dfrac{1}{(x^2+1)(x^2+2)} = \dfrac{Ax+B}{x^2+1}+\dfrac{Cx+D}{x^2+2}$

$$\Rightarrow \frac{(Ax+B)(x^2+2)+(Cx+D)(x^2+1)}{(x^2+1)(x^2+2)}$$

صورت کسر: $Ax^3+2Ax+Bx^2+2B+Cx^3+Cx+Dx^2+D=(A+C)x^3+(B+D)x^2+(2A+C)x+(2B+D)=1$

$$\Rightarrow \begin{cases} A+C=0 \\ 2A+C=0 \end{cases} \rightarrow A=C=0 \qquad \begin{cases} -B-\cancel{D}=0 \\ B+D=0 \\ 2B+\cancel{D}=1 \end{cases} \quad B=1 \quad D=-1$$

$$\Rightarrow \int\frac{dx}{(x^2+1)(x^2+2)} = \int\left(\frac{1}{x^2+1}-\frac{1}{x^2+2}\right)dx = \tan^{-1}x - \tfrac{1}{\sqrt{2}}\tan^{-1}\tfrac{x}{\sqrt{2}} + C$$

$a=\sqrt{2}$

$g(x)$ $\quad R(x)$

$\dfrac{f(x)}{g(x)}$

$$\begin{array}{l|l} f(x) & g(x) \\ \hline & S(x) \\ \overline{R(x)} & \end{array}$$

$\deg(R(x)) < \deg(g(x))$

$$\int \frac{f(x)}{g(x)} = \int S(x) + \boxed{\frac{R(x)}{g(x)}}$$

$$\int \frac{x^4}{x^3+x^2+x+1}\,dx \qquad \frac{x^4}{x^3+x^2+x+1} = x-1 + \frac{1}{x^3+x^2+x+1}$$

$$\begin{array}{l|l} \cancel{x^4} & x^3+x^2+x+1 \\ \hline \cancel{-x^4-x^3}-x^2-x & x-1 \\ \hline \cancel{-x^3}-x^2-x & \\ \cancel{x^3+x^2+x}+1 & \\ \hline 1 & \end{array}$$

$$\frac{1}{x^3+x^2+x+1} \Rightarrow x^3+x^2+x+1 = x^2(x+1)+(x+1) = (x+1)(x^2+1)$$

$$\frac{1}{x^3+x^2+x+1} = \frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)(x+1)}{(x+1)(x^2+1)}$$

صورت کسر: $Ax^2+A + Bx^2 + Bx + Cx + C = (A+B)x^2 + (B+C)x + (A+C) = 1$

$A+B=0$

$\underbrace{}_{B=-A}$

$\cancel{B}+C=0$ ($-A$)

$A+C=1 \Rightarrow \begin{cases} -\cancel{A}+C=0 \\ \cancel{A}+C=1 \end{cases} \rightarrow 2C=1 \rightarrow C=\frac{1}{2}$

$A=C=\frac{1}{2} \Rightarrow B=-\frac{1}{2}$

$$\int \frac{1}{(x+1)(x^2+1)}\,dx = \int\left(\frac{\frac{1}{2}}{x+1} + \frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}\right)dx = \frac{1}{2}\left\{\int \frac{1}{x+1}\,dx - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\right\}$$

$$= \frac{1}{2}\left\{\ln|x+1| - \frac{1}{2}\ln(x^2+1) + \tan^{-1}x\right\}$$

جواب نهایی: $\displaystyle\int \frac{x^4}{x^3+x^2+x+1}\,dx = \int\left(x-1+\frac{1}{x^3+x^2+x+1}\right)dx$

$$= \frac{1}{2}x^2 - x + \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\tan^{-1}x + C$$